Hale School

Mathematics Specialist

Test 3 --- Term 2 2017

Vectors in 3D

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Name: _____

Instructions:

- **CAS calculators are allowed**
- **External notes are not allowed**
- **Duration of test: 45 minutes**
- **Show your working clearly**
- **Use the method specified (if any) in the question to show your working (Otherwise, no marks awarded)**
- **This test contributes to 7% of the year (school) mark**

Question 1 **(8 marks: 2, 2, 2 and 2)**

Consider points A (1 , 3 , 5) , B (-7 , 5 , 1) and C (3 , -2 , 4).

(a) Determine the vectors **AB** and **AC**.

(b) Determine the vector equation of line containing points A and B.

(c) Find a vector perpendicular to both **AB** and **AC**.

(d) Find the Cartesian equation of the plane passing through A, B and C.

Question 2 (8 marks: 3, 5)

The vector equations of lines L and M are given by

$$\vec{r}_L = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \quad \text{and} \quad \vec{r}_M = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} \quad \text{respectively.}$$

(a) Show that the lines do not intersect.

(b) If P and Q are points on L and M, the two lines are closest when PQ is perpendicular to both line L and line M. Find the closest distance between the two lines accurate to 2 decimal places.

Question 3 **(5 marks: 3, 2)**

At 5am, hot-air balloons A and B leave their home bases located at $-10\mathbf{i}+40\mathbf{j}+0.2\mathbf{k}$ and $15\mathbf{i}+60\mathbf{j}+0.05\mathbf{k}$ with constant velocities $\mathbf{v}_A = 10\mathbf{i}+40\mathbf{j}+\alpha\mathbf{k}$ and $\mathbf{v}_B = 5\mathbf{i}+\beta\mathbf{j}+0.02\mathbf{k}$. Measurements are in km and km/hr.

(a) Find the values of α and β given that the two balloons collide.

(b) State the time and position of the collision.

Question 4 **(8 marks: 2, 2, 2, 2)**

(a) In each case below, state whether the system of equations has a unique solution, no solutions or an infinite number of solutions. State the geometric relationship between the planes in each case.

i)
$$\begin{aligned} x + 2y + z &= 7 \\ 3x + 6y + 3z &= 11 \\ 2x - 3y - z &= 4 \end{aligned}$$

ii)
$$\begin{aligned} x + y - z &= 4 \\ 3x + 5y + 2z &= 7 \\ 2x + 4y + 3z &= 3 \end{aligned}$$

(b) Consider the system of equations in x, y and z with augmented matrix,

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 4 \\ 0 & 5 & -3 & 12 \\ 0 & 0 & p^2-4 & p-2 \end{array}\right]$$

i) Find the solution to the equations when $p = -1$

ii) State the value(s) of p for which there are no solutions to the system of equations.

Question 5 (7 marks: 2, 5)

Given the sphere with equation $\left|\vec{r}-\begin{pmatrix}-1\\2\\3\end{pmatrix}\right|=4$ and the plane with equation $x+2y+z=24$

(a) Find the equation of the straight line passing through the centre of the sphere that is perpendicular to the given plane.

(b) Find the exact distance between the plane and the sphere.

Question 6 (11 marks: 3, 3, 3, 2)

A particle moves along a path described by the vector function $\mathbf{r}(t) = (3+4\cos t)\mathbf{i} + 2\sin t\,\mathbf{j}$ for $t \geq 0$.

(a) Determine but do not simplify the Cartesian equation of the path.

(b) Show that the speed of the particle at time t is given by $\sqrt{4+12\sin^2 t}$.

(c) Determine the location(s) of the particle when it has minimum speed.

(d) Write down and evaluate, to 2 decimal places, an integral that will determine the distance travelled by the particle in the first 5 seconds.